# Second-Best Combinatorial Auctions – The Case of the Pricing-Per-Column Mechanism

Dirk Neumann, Björn Schnizler, Ilka Weber, Christof Weinhardt
*Institute of Information Systems and Management (IISM)*
*Universitaet Karlsruhe (TH)*
*[neumann, schnizler, weber, weinhardt]@iism.uni-karlsruhe.de*

## ABSTRACT

*One of the main contributions of classical mechanism design is the derivation of the Groves mechanisms. The class of Groves mechanisms are the only mechanisms that are strategy-proof and more importantly allocative efficient. The VCG mechanism retains its properties for combinatorial allocation problems. From a computational perspective the VCG has to solve two problems: (1) the winner-determination (2) the determination of the prices. However, both problems are complex (NP-hard), when complementarities are present. The Pricing-Per-Column (PPC) auction is another approach to solve the combinatorial allocation problem. In essence, it applies the Vickrey principle to any possible combination of goods and determines the overall winning bids. PPC is computationally less demanding, however, it can be shown that PPC is not necessarily efficient. Apparently, solving the tension between computational and game-theoretic properties is a challenging task in mechanism design. Engineering auctions suggests to lower requirements upon the auction. In this paper the evaluation of the PPC concerning approximate efficiency is presented - in an analytical and simulative evaluation the PPC is compared to the VCG and it is shown that the efficiency losses incurred by the PPC mechanism are very small.*

## 1. INTRODUCTION

Auctions have become an important coordination mechanism for supporting negotiations in distributed systems like the Internet. Their popularity increased through a multitude of online markets such as eBay, Yahoo!, and Amazon. These platforms list millions of items for sale and attract a multitude of users. They usually make use of traditional auction mechanisms such as variants of the English auction for trading single and multiple homogenous goods.

However, these auction mechanisms fail when a set of heterogeneous goods are traded simultaneously. One reason for this lies in the dependencies between goods in terms of complementarities and substitutes. In such cases, the valuation for a single good is influenced by additional allocations of other goods. For instance, suppose a bidder requires a travel package including a flight, a hotel, and a rental car. Suppose the participant has a positive valuation for the whole bundle. An allocation of the car without the flight is, however, useless for him. Flight, hotel, and rental car supplement each other and thus are complementarities. The valuation for the whole bundle is higher than the sum of the valuations for the single items. Goods may also be substitutes, i.e. they may be replaced by similar items. Suppose the bidder is willing to pay 400€ for the hotel at the destination. If he gets allocated two rooms simultaneously, his valuation for both rooms is still 400€. In this case, the utility of getting allocated two similar items is not higher than the utility of a single good.

Auction mechanisms supporting complementarities and substitutes are called combinatorial auctions. Combinatorial auctions are important in many real-world problems such as for auctioning spectrum licenses [12], allocating airport time slots [17], or procuring transporting services [19]. Although combinatorial auctions can be approximated by multiple single-item auctions, this often results in inefficient outcomes [2].

Efficient mechanisms in the presence of complementarities are of the Groves family. Known as the Vickrey-Clarke-Groves (VCG), the mechanism provides a dominant-strategy solution to report preferences truthfully even in the presence of complementarities. The peculiarity of the VCG mechanism is that the price an individual bidder has to pay depends on the bids of other bidders.

Despite its theoretical soundness and elegance, the VCG incurs severe drawbacks when complementarities are present: From a computational point of view, the VCG is hard to solve with an increasing number of goods and participants. The winner determination problem in combinatorial auctions is NP-hard, as it is an instance of the set packing problem (SPP) [3]. In the VCG, this problem has to be solved $N+1$ times, once with all participants and then $N$ times more with each of the $N$ participants removed from the allocation. In addition to this tractability issue, combinatorial auctions are often hampered by the fact that even the preference elicitation problem is hard to solve [13]. For instance, if three goods are available, say A, B and C, the VCG mechanism requires the definition of values for {A}, {B}, {C}, {AB}, {AC}, {BC}, and {ABC}. If the number of goods increases, the number of feasible sets that need to be valued by the bidders increases over-proportionally.

To alleviate this preference elicitation problem, recent work suggests iterative auction formats, e.g. iBundle [14] or Simultaneous Ascending Auction with Package Bidding [1]. The informational requirements upon iterative mechanisms are relatively mild, considering the fact that the mechanisms need only information about the valuation for the bundle the bidder values most. Instead of weighing all possible combinations, one single value is sufficient. Beside this argument, it is often referred to the theoretical result from single item auctions, where iterative auctions yield higher revenue for the seller than one-shot auctions.

The line of argumentation follows this intuition: if valuations are affiliated, iterative mechanisms will be more desirable than one-shot mechanisms. Although those arguments are convincing, the conclusion should not be that iterative mechanisms are always preferable to one-shot mechanisms. If, for instance, the auction is fully automated by bidding agents, the preference elicitation problem may be tractable for relatively small number of goods. Since immediacy is of concern, iterative auctions are inferior to one-shot auctions. In this case, it appears to be reasonable to employ one-shot auctions. But here the problem arises: which one to use? As aforementioned, the VCG mechanism faces the problem of its NP-hardness when complementarities are involved.

In this paper, we will analyze the Pricing-Per-Column (PPC) mechanism as an alternative to the VCG [4]. Although it cannot solve the preference elicitation problem, which is inherently associated with one-shot combinatorial auctions, it alleviates the computational complexity of the price determination problem considerably. It will be shown that the efficiency losses incurred by the PPC mechanism are very small.

The remainder of the paper is structured as follows: Firstly, the requirements the auction should satisfy are listed. Subsequently, the Pricing-Per-Column auction is introduced as a candidate solution to those design requirements. Then, the auction is analytically and numerically evaluated and compared to the VCG. The paper concludes with a summary and an outlook for future research.

## 2. Requirements on the Design

Any design task begins with the elicitation of the requirements the design artifact must satisfy. This paper addresses a combinatorial auction format that is suitable for a certain class of domains. For simplicity, it is convenient to focus on one concrete scenario and derive the requirements upon the auction format and upon the quality of the results this auction produces.

Consider a transportation scenario, where several trucks transport freight from one depot to another. Demand will be conveyed to a central department, which contracts out the transportation jobs to the subsidiary depots. The depots manage their truck fleet decentralized and bid on the single jobs. For instance, a depot may bid 1.500 € for transporting 1,000 kg from Zurich to Prague. Furthermore, transportation jobs are characterized by complementarities among the jobs. Suppose the example from before: the depot bids 1.500 € for a job from Zurich to Munich, and, in addition 1.500 € from Prague to Warsaw. If the depot is allocated both jobs, the depot may raise its bid from 3.000 € to 4.500 €. The reason for this increasing value for the bundle stems from the synergies that can be realized. Instead of returning from Munich back to the depot after having completed the first job, the second job can be undertaken, saving the costs incurred by returning to the depot. This simple scenario imposes several requirements upon the auction mechanism and upon the outcome the auction achieves.

Concerning the mechanism, the scenario depicts a situation, where a single sided mechanism is necessary that can cope with combinatorial bids. More precisely, the requirements on the mechanism are as follows:

- *Single-sided mechanism:* The central entity aggregates demand and contracts them out to the depots. This means, only the central entity is the seller (or issuer) of jobs without competitors, while the depots compete against each other.
- *Language includes combinatorial bids:* The depots often demand a combination of jobs as a bundle to realize synergies. As such, transportation jobs are complementarities, i.e. participants have super-additive valuations for the jobs, as the sum of the valuations for the single job is less than the valuation for the whole bundle $(v(A)+v(B) \leq v(AB))$. Suppose a depot bids for the bundle {(Zurich, Munich) and (Prague, Warsaw)}. If one component, e.g. the first job, is not allocated to him, the remaining bundle (consisting of the last job) has a decreased value for him since no synergies can be realized. In order to avoid this exposure risk (i.e. receiving only a subset of the bundle), the mechanism must allow for bids on bundles. Furthermore, the depots may also want to submit more than one bid on a bundle but many that are excluding each other. In this case, the jobs of the bundles are substitutes. This means that the buyer has sub-additive valuations ($v(A)+v(B) \geq v(AB)$) for the jobs. For instance, a depot is willing to pay a high price for a transportation job during the day

and a low price if the job is done at night. However, this transportation job can be done only once. As such, the market mechanism must support XOR[1] bids to express substitutes.

- *Clearing and pricing rules that exploit the full-range of the language:* Furthermore, clearing and pricing rules have to be designed that (1) impute a desirable allocation (allocative efficient) and (2) make usage of all information of the bidding language.
- *One-shot Process:* Due to the dynamic nature, the bidding process will be automatically conducted by agents. To delimit the timing of an auction and to confine the strategic complexity of the bidding agents, the auction needs to be one-shot.

Concerning the outcome of the auction, the scenario suggests the following properties that the mechanism should satisfy:

- *Allocative efficiency*: An allocative efficient allocation of jobs maximizes the sum of individual profits. Since, the depots and the central department belong to one single organization, the maximization of all profits results in the maximization of the organization's profit.
- *Incentive Compatibility:* Achieving an allocative efficient allocation of the jobs requires that all depots truthfully report their valuations. The auction should thus induce incentive compatibility, i.e. all depots report their preferences truthfully in equilibrium. In the optimal case, truth-telling is a dominant strategy, since the depots have no incentive to untruthfully report their preferences in order to increase their individual utility.
- *Individual Rationality:* Another requirement is that the depots voluntarily join the auction. This in turn requires that the profit the depots derive from participation is greater or equal than before, since the depots would otherwise decide to opt out.
- *Budget Balance:* A mechanism is said to be strictly budget balanced if the amount of prices sum up to 0 over all depots. In this case neither are funds removed from the system nor is the system subsidized from outside. Strict budget balance is an important property since the resource allocation can be performed at no costs. In case the mechanism runs a deficit, the organization has to subsidize the deficiary depots. Such a situation cannot be sustained for a longer time period [8, 15].
- *Computational tractability:* Computational tractability considers the complexity of computing the outcome of a mechanism from the depots' strategies. With an increasing size of bids, the allocation problem can become very demanding. Thus, computational constraints may delimit the design of the proper auction mechanism [9, 10].

# 3. VCG and PPC Mechanisms

For modeling the auction mechanisms for the transportation scenario, a widely used private value model is employed, where depots have incomplete information about the preferences of the other depots [c.f. 8]. There are $N$ depots and $\Theta^i$ defines the set of all possible types for depot $i$. This type $\theta_i \in \Theta^i$ for depot $i$ specifies the preferences of $i$ and also $i$'s information about other depots.

A mechanism $\mathcal{M}$ is defined as the available bids and the rules how to resolve them; that is: mechanism $\mathcal{M}$ is a pair $(M, h(M), t_1(M), t_2(M), ..., t_N(M))$, where $M = M^1 \times M^2 \times ... \times M^N$ and $h{:}M \rightarrow A$ and $t_i{:}M \rightarrow \Re$ . The term $M^i$ refers to the message space of $i$, whereas $h$ denotes the allocation function that computes who gets what and $t_i$ denotes the payments of each depot. For one-shot (or so-called direct) mechanisms the message space simplifies to $M^1 = \Theta^1$. In such a one-shot mechanism, the strategy of depot $i$ is $\hat{\theta}_i \in \Theta^i$. The reported type $\hat{\theta}_i$ can equal the true type, but can also be another type. Given a joint strategy $\hat{\theta} = (\theta_1, \theta_2, ..., \theta_N)$, the outcome generated by $\hat{\theta}$ is denoted by $h(\hat{\theta})$ and $t_i(\hat{\theta})$. Depots are assumed to be risk neutral and have quasi linear utility functions. That is, utility function of depot $i$ is $u_i((h,t_i), \theta_i) = v_i(h, \theta_i) + t_i$.

## 3.1 The VCG mechanism

One of the most prominent mechanisms in mechanism theory are the so-called VCG (Vickrey-Clarke-Groves), pivotal, or Clarke mechanisms. What make the VCG mechanism powerful in mechanism design are the nice properties associated with it, which will be presented in section 4.

The allocation rule $h$ of a VCG mechanism is specified by the maximization of the reported valuations. Generally, this is denoted by

$$h^*(\hat{\theta}) = \arg\max_{h \in X} \sum_i v_i(h, \hat{\theta}_i) .$$

The transfer rule (in this case of a pivotal mechanism) for depot $i$ amounts to

$$t_i(\hat{\theta}) = \sum_{j \neq i} v_j(h^*_{-i}(\hat{\theta}_{-i}), \hat{\theta}_j) - \sum_{j \neq i} v_j(h^*, \hat{\theta}_j).$$

[1] A XOR B (A ⊗ B) means either A or B but not both

The interpretation of the transfers is instructive [8]: if depot $i$'s presence does not make a difference in the maximizing problem (viz. agent $i$ is not part of the optimal allocation), the payments are zero. Otherwise $i$'s presence is pivotal, as the social welfare, i.e. the sum of all agents, is affected by the participation of depot $i$. The payments exactly reflect the loss in valuation of the other depots, which is incurred by the participation of depot $i$. The VCG mechanism incorporates the marginal impact on the other valuations by the announcement of $\hat{\theta}_i$ into the payment function internalizing this external effect. At the bottom-line the individual depot is thus forced to consider also social welfare when making his choice.

The VCG mechanism can also be applied to combinatorial allocation problems. The mechanism can then be formalized as follows: Let $G$ be a set of single items and $s_i \subseteq G$ be a bundle which can be allocated to the participants. An efficient allocation can be computed as follows:

$$S^* = \underset{S=\{s_1 \dots s_I\}}{\arg\max} \sum_i v_i(s_i) \quad (3)$$

$$\text{s.t. } s_i \cap s_j = \phi \,, \; \forall i, j \,. \quad (4)$$

The objective (3) maximizes the total utility of the allocation. The first constraint (4) ensures, that no item is allocated to more than one participant. Let $V^*$ denote the total value of the allocation including all participant and let $(V_{-i})^*$ denote the value of the allocation without participant $i$. The payment rule for a participant $i$ can be calculated as the difference between the reported valuation and its impact on the allocation:

$$p_{VICK,i}(s_k) = v_i(s_k) - (V^* - (V_{-i})^*). \quad (5)$$

**Example 1: The VCG mechanism**

Consider there are four depots A, B, C and D competing for three transportation jobs G, H, and T. The valuations for the jobs are given by Table 1. The depots would report these valuations truthfully.

**Table 1: Valuation Matrix**

| Jobs / Depot | {G} | {H} | {T} | {G,H} | {H,T} | {G,T} | {G,H,T} |
|---|---|---|---|---|---|---|---|
| A | 10 | 30 | 40 | 65 | 10 | -20 | -30 |
| B | 5 | -10 | 30 | -30 | 40 | 80 | 80 |
| C | -10 | 70 | 60 | 40 | -20 | 45 | 10 |
| D | 5 | 40 | 35 | 40 | 60 | -30 | 50 |

The VCG mechanism would allocate jobs {G, T} to depot B and {H} to depot C, since this maximizes the sum of all individual valuations. The payments would be for depot A and D 0, as they do not enter the allocation. Depot B would have to pay its valuations minus the difference of the sum of all valuations and the sum of all valuations in the allocation except depot B. The optimal allocation without depot B would be {G,H} to depot A and {T} to depot C accruing a total valuation of 60 + 65 = 125. Hence, depot B faces a payment of 80 – (150 -125) = 55. Analogously, depot C is subject to a payment of 40.

## 3.2 The PPC mechanism

The PPC mechanism [4] distinguishes itself from the VCG mechanism by the payment function. Thus, the allocation rule $h$ of PPC mechanism is analogous to the VCG mechanism

$$h^*(\hat{\theta}) = \underset{h \in X}{\arg\max} \sum_i v_i(h, \hat{\theta}_i) \quad (3)$$

The transfer rule of the PPC mechanism for depot $i$ is denoted by the valuation that would occur if depot $i$ is not present keeping the distribution of bundles constant. This can be easily explained by referring to Table 1. If it is assumed that the depots reveal their valuations truthfully, depot B and C would still receive the allocation of {G,T} and {H}, respectively. The price depot B has to pay is according to the PPC scheme, the highest valuation for {G,T} if B would be present. Accordingly, the columns of the allocation are fixed and the second price is used. Hence, B has to pay the second highest price of the column, which amounts to 45 reported by depot C.

This intuitive transfer rule is specified by:

$$t_i\left(\hat{\theta}\right) = -\underset{j \neq i}{\arg\max}\, v_{ik^*}(h, \hat{\theta}_i) \quad (4)$$

for all winning bids assumed that there are at least two bids on the column and 0 otherwise. Furthermore, only bids that are lower or equal the winning bid in a column are considered. Note that k* refers to the column index, which is part of the winning allocation.

In essence, both the VCG and the PPC mechanism differ in their payment rule definition and the prices associated with it. Since the payments depots have to pay have an influence on their bidding behavior, the results are likely to be different.

## 4. Evaluation

The evaluation of the mechanisms will be twofold. In the first part, an analytical approach will be conducted. The analytical approach, nonetheless, is not sufficient enough to shed light into the question, which mechanism is ultimately to be preferred. Hence, a numerical simulation will explore those questions, for which the analytical approach is silent.

### 4.1 Analytical Evaluation

The theory of mechanism design provides a theoretical toolbox for designing institutions with a particular emphasis on incentives [11]. The problem of designing a mechanism, i.e. game form, is to implement a mechanism $(M, y)$, such that the equilibrium outcome satisfies a particular social choice function.

A social choice function *f(θ)* is denoted as a pair *(h(θ),$t_1$(θ), $t_2$(θ),..., $t_N$(θ) )*. That is, given any preference profile of the depots, the social choice function chooses one allocation of jobs and corresponding payments. In essence, the task of the mechanism designer (in our example, the managers of the transportation firm) would be ideally to choose the allocation that maximizes valuations of the society (i.e. all depots of the firm). Such a social choice function, which satisfies $h \in \arg\max_{h \in H} \sum_{j=1}^{N} v_j\left(h(\theta), \theta\right)$ is denoted to be efficient.

It can be shown that it is not sufficient to maximize the sum of reported preferences $\hat{\theta}$ to implement an efficient social choice function. The depots will attempt to manipulate their reports such that the can extract more benefit from the mechanism. Payments are needed in order to incentivize depots to report their valuations truthfully at all time, regardless of what the other depots bid. This refers to strategy proofness, which denotes that truth-telling is a dominant strategy: For all $i \in N$, the individual benefit from truth-telling is at least as high as from lying $u_i\left(\theta, \theta_i, h, t\right) \geq u_i\left(\theta_{-i}, \theta_i\right)$.

Mechanism design has found out that one class of mechanisms, the so-called Groves mechanisms, is strategy proof. A Groves mechanism is defined by the following (i) allocation and (ii) payment rule.

(i) $h \in \arg\max_{h \in H} \sum_{j=1}^{N} v_j\left(h(\theta), \theta\right)$

(ii) $t_i\left(\hat{\theta}\right) = x_i\left(\theta_{-i}\right) + \sum_{j \neq i} v_j\left(h(\theta), \theta_j\right)$

where function $x_i : \times_{j \neq i} \Theta_j \rightarrow \Re$ independent of $\theta_i$ is.

**Theorem 1: Groves mechanisms (Groves 73):** *Groves mechanisms are strategy proof.*

The proof that Groves mechanisms are strategy proof and, hence, lead to an efficient allocation of the jobs can found at [6].

**Theorem 2: VCG and Groves mechanisms:** *The VCG mechanism is a special case of the Groves mechanism*

It is trivial to show that the VCG mechanism is part of the group of the Groves mechanisms with an $x_i = \max_{h \in H} \sum_{i \neq j} v_j\left(h, \theta_j\right)$.

**Theorem 3: Groves uniqueness (Green and Laffont 77):** *The class of Groves mechanisms are the only mechanisms, which are efficient and strategy proof.*

The proof can be found at [5].

Groves mechanisms lead to strong economic properties. However, from a computational point of view, Groves mechanisms may be intractable [15]. The PPC mechanism has been introduced to alleviate the computational problem. Nonetheless, the properties of the PPC have not been analyzed. From theory, it is known that only Groves mechanisms implement an efficient social choice function in dominant strategies. As such, it is sufficient to show whether the PPC mechanism belongs to the class of Groves mechanisms.

**Theorem 5: PPC Inefficiency:** *The PPC mechanism is not strategy proof.*

To show theorem 5, all that is necessary is to prove that the PPC mechanism is not a Groves mechanism. Since Groves and PPC mechanisms have an identical allocation rule, the attention can be restricted to the payments rule. In essence, it needs to be demonstrated that the PPC payment function cannot be represented as a special Groves payment. The intuition that this cannot be made stems from the fact that the individual Groves-payment does not depend on that individual's reported valuations. In the PPC the payments, on the contrary, depend on the individual reports, since the reports can determine the division of the goods into its constituent bundles (e.g. the columns) and hence the price. For example a depot that is not part of the optimal allocation, in Example 1 the bidder A, can enter the allocation by overstating his valuation for {G, H} by 40 adding up to 105 (note that A's true valuation for this bundle is 65). In this case, A is now part of the allocation, while C receives {T}. A's payment amounts to the second highest price on the bundle {G, H}, which is 40. By this overstatement, depot A can realize a gain of 65 – 40 = 25, which is positive. The full proof can be found in the Appendix.

Theorem 5 is in combination with theorem 1 and 3 strict in its message: the PPC mechanism cannot implement an efficient social choice function in dominant strategies. Manipulating the reports can tremendously bias the allocation. Nonetheless, theorem 5 does not reveal any information about how the depots would manipulate their valuations, if it all. In our fixed example, it is rational for depot A to overstate his valuation, knowing the reports of all other depots. The PPC mechanism, however, is sealed in nature. Hence, the depots have no clear information about the other reports at all. Overstating the own valuation is dangerous, as the depots run the risk of suffering a loss. For instance, if C's true valuation for bundle {G ,H} is 70 instead of 40. If depot A manipulate his bid to 105, he would receive {G ,H}, which is worth 65 for him, while he pays 70, resulting in a loss. In essence, other depots taking part in the mechanism are alleviated the manipulation behavior.

It is the claim of this paper that depots reveal their valuations approximated truthfully, as the (game-theoretic) optimal strategy is too complex to determine. Referring to Simon [20], humans (i.e. depots in this context) will likely adopt very simple heuristics. The simplest heuristic would be to report the valuation truthfully. A numerical simulation will be used to evaluate this claim.

## 4.2 Numerical Evaluation

In the PPC, depots (as bidders) may have an incentive to misrepresent their valuations in order to gain higher utility. This potential gain will be measured by performing a stochastic simulation. The VCG and the PPC are implemented in a Java based simulation environment[2]. For solving the winner determination problem, CPLEX 9.13 is used.

As reports, buyers submit a set of XOR concatenated bids on all possible bundle combinations. For instance, in a scenario with three different jobs (A, B, C), buyers bid on the bundles {A}, {B}, {C}, {AB}, {BC}, and {ABC}. The valuations for each of the bundle bids are drawn from a uniform distribution. Each scenario is repeated 50 times with different initializations; the results are averaged.

Only simple misrepresentations by α% are considered, where β% of the buyers increase their reported values by α%. Instead of observing only symmetric Nash-equilibria as in the analysis by [16], where participants either misrepresent their valuation price by 0 or β%, the ratio of misrepresenting buyers to the total number of participants is also varied. A ratio of β=50%, for instance, denotes that 50% of the buyers misrepresent their valuations by α%, while the other 50% report truthfully.4

By exploring the joint strategy space (i.e. varying the share of misrepresentative and truthful depots as well as the percentage of misrepresentation) the average utility gain of manipulating depot can be measured. This analysis provides information on whether or not the total utility of depots can be improved through manipulation. The utility, which depots can gain by manipulation, only depends on the prices depots have to pay and is thus calculated as:

$$UG = \sum_{s_j \in S} \sum_{i \in \hat{I}} \hat{p}_i(s_j) - \sum_{s_j \in S} \sum_{i \in \bar{I}} \bar{p}_i(s_j),$$

where $\hat{I}$ is the set of buyers who are part of the allocation in the treatment with manipulation participants and $\bar{I}$ is the set of successful buyers in the truthful treatment. For a better comparability of the results, the utility gain UG is further specified as the percentage of the truthful scenario.

In the first setting with 50 buyers and 5 different jobs (i.e. 31 different bundles), the utility gain of manipulating depots are shown in Figure 1 and Figure 2.

**Figure 1: Utility gain using PPC, 10 buyers, 5 jobs, 31 bundles**

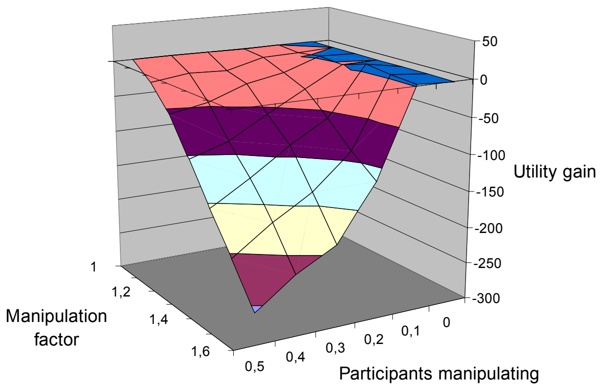
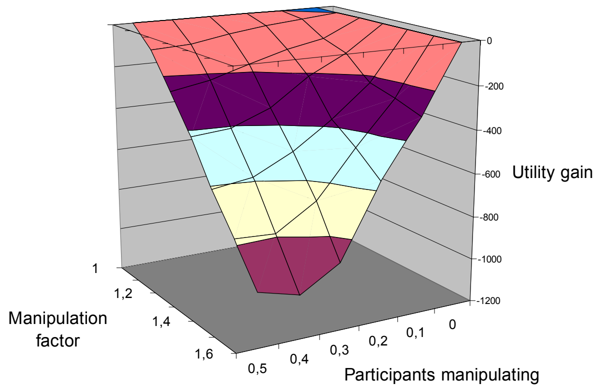



**Figure 2: Utility gain using VCG, 10 buyers, 5 jobs, 31 bundles**

[2] See http://www.iw.uni-karlsruhe.de/jcase for details.

[3] CPLEX is a mathematical optimization engine for solving linear programs (http://www.ilog.com/ ).

[4] This restriction is being made, as the results above 50 % manipulation suggest a tremendous decrease in individual utility and are thus left out.




In both cases – using the VCG and the PPC – buyers mostly do worse by overbidding, i.e. revealing a higher value than their true valuation. The VCG penalizes all manipulation attempts by a utility loss. In the PPC, utility can be gained only when few depots manipulate. The highest utility gain (9.5%) is achieved when 10% of the buyers bid 140% of their valuation. This simulation result suggests that the PPC mechanism – compared to the VCG – results in nearly equal overall utility and thus may have accurate incentive properties. With a varying number of buyers (to 50 and 3), nearly the same results are observed.

In a second setting, the number of 3 different jobs and thus 7 different bundles were used. Figure 2 shows the result of the simulation. In both cases, no utility can be gained by overbidding. As seen in the first scenario, the VCG again penalizes manipulation more than the PPC. The simulation shows that the PPC nearly achieves the same properties than the VCG.

**Figure 3: Utility gain using PPC, 10 buyers, 3 jobs, 7 bundles**

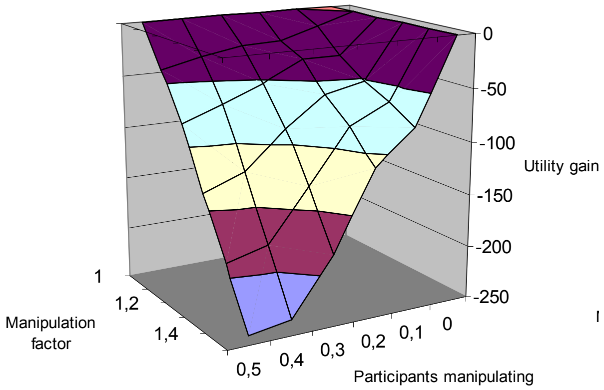
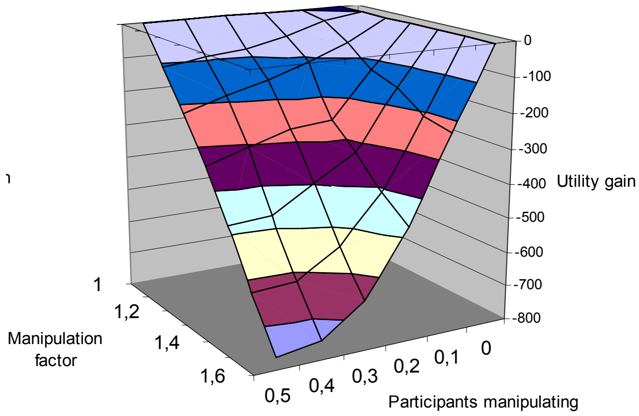



**Figure 4: Utility gain using VCG, 10 buyers, 3 jobs, 7 bundles**




The first two scenarios suggest that the PPC works fairly well with an average number of jobs (scenario 1) and is nearly comparable to the VCG with a very low number of jobs (scenario 2). Obviously, a higher number of jobs lead to a lower incentive compatibility. Hence, the number of jobs is increased in a third scenario to emphasize this effect. The third scenario comprises 7 different jobs and as such 127 different bundle combinations. Figure 3 and Figure 6 depict the results for the PPC (left) and VCG (right). The increasing incentive compatibility effect is emphasized. If a small number of participants overbid, they can achieve a higher utility gain than in the first scenario. Nevertheless, the highest utility is only X% if Y% of the participants overbid their valuation by Z%.

In summary, the simulations have shown that it is reasonable to believe that participants will not strongly deviate from their truth valuations. Although participants' average utility gain can be improved through manipulations if the number of jobs is high enough, the participants increasingly also risk not being executed in the auction. This risk actually increases the more participants use manipulation. The simulation result suggests that the PPC has accurate incentive properties resulting in fairly mild allocative efficiency losses.

**Figure 5: Utility gain using PPC, 10 buyers, 7 jobs, 127 bundles**

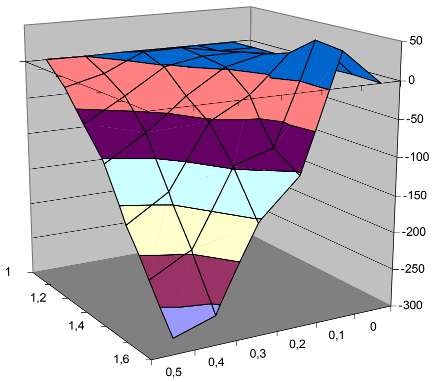
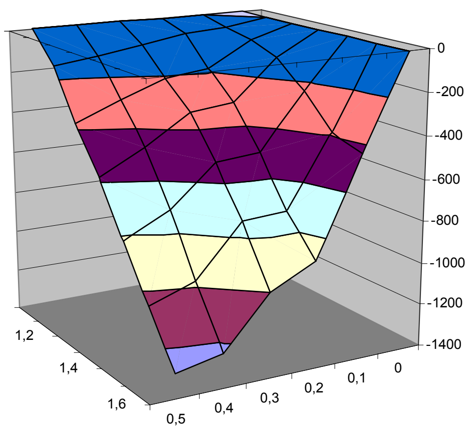



**Figure 6: Utility gain using VCG, 10 buyers, 7 jobs, 127 bundles**




## 5. Conclusion

Combinatorial mechanism design is particularly difficult, as there is a tension between economic characteristics, on the one hand, and computational properties, on the other hand. From an economic point of view, the VCG mechanism is the only direct mechanism that achieves (i) an efficient allocation of goods, (ii) voluntary participation of the bidders and (iii) incentive-compatibility. Nonetheless, the VCG mechanism lacks tractability, as the winner determination problem (i.e. allocation rule), and also the N+1 payment computations, are NP hard. In addition to those problems, the VCG mechanism needs the preference values for all possible combinations of the goods from each bidder to retain its desirable economic properties. This so-called preference elicitation problem is also NP-hard for any participating bidder. Hence, state-of-the-art mechanism design has turned its attention from direct, one-shot auctions to iterative auctions. In iterative auctions, the bidders need only to bid on the bundle that provides them with the highest utility.

In this paper, it is argued that it can make sense to apply direct mechanisms in favor of iterative mechanisms. The PPC mechanism is a very simple mechanism, which eases the payment computations. As classical mechanism design suggests, the PPC mechanism is not strategy proof any more. Nonetheless, classical mechanism design cannot state how the bidders will behave when exposed to this mechanism. In our numerical simulation, we show that deviating from the true valuations does not improve the individual utility if the number of competing bidders is sufficiently high with respect to the available goods. This property is lost, if the number of available goods is increased.

This leads to the conclusion that when the size of the auction is very large, strategizing does not pay off. Competition drives the bidders to reveal their true valuations. As the number of bidders increase relatively to the number of goods, the results of PPC mechanism converge to the ones of the VCG mechanism. In that case, the PPC mechanism can be seen as a kind of second-best mechanism. For decision support systems that guide the bidders in forming their bids, this has the implication that the emphasis is shifted from devising bidding strategies towards preference elicitation.

To fully exploit the merits of the PPC mechanism, several problems need to be overcome. Firstly, the preference elicitation problem remains complex, as all valuations need to be reported. Hence, the mechanism needs bidding support, when the number of goods becomes large. Even though bidding support can be reduced to extracting the true valuations, this can become cumbersome when the valuations are not exactly known. In this case, bidding support may need to apply some heuristics to approximate the valuations. The effects on the mechanism result when estimation errors are present are widely unknown. Secondly, iterative mechanisms yield higher revenues when valuations are affiliated. Hitherto, the issue of affiliation has been left out of the analysis. Thirdly, experiments with few goods are needed to verify the conjectures made in this paper.

## Appendix

**Proof of Theorem 5 (PPC Inefficiency)**

All we have to show is that the payment function of the PPC cannot be represented as the payment function of a Groves mechanism. Based upon the aforementioned definitions, that is $t_i^{PPC} \notin t_i^{Groves}$.

$$\begin{aligned} t_i^{PPC}(\theta) &= -\max_{j \neq i} v_{jk*} \\ &= -\max_{j \neq i} v_{jf(h,i)} \\ &= \left( -\max_{j \neq i} v_{jf(h,i)} + \sum_{j \neq i} v_j\left(h(\theta), \theta_j\right) \right) - \sum_{j \neq i} v_j\left(h(\theta), \theta_j\right) \\ &= x_i - \sum_{j \neq i} v_j\left(h(\theta), \theta_j\right) \end{aligned}$$

For a Groves mechanism, the term $x_i$ is independent of the individual i's reports. If the PPC mechanism had been a Groves mechanism, $x_i = x_i(\theta_{-i})$ would hold.

For the PPC $x_i$ was defined as

$$-\max_{j \neq i} v_{jf(h,i)} + \sum_{j \neq i} v_j\left(h(\theta), \theta_j\right)$$

The first term is identical with the payments of the PPC, the second aggregates the valuations of all depots within the efficient allocation without considering individual i's valuation. For the first term $-\max_{j \neq i} v_{jf(h,i)}$

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